

$$s = s(r) = \int^r f(q) dq, \quad (f > 0), \quad (6)$$

and the coefficient $F = F(s)$ in (5) is the function¹

$$F = \frac{3}{4} f'^2 / f^4 - \frac{1}{2} f'' / f^3. \quad (7)$$

It is understood that $f = f(r)$ and its derivatives with respect to r must, in (7), be expressed as functions of s by means of the inverse of the substitution (6).

Suppose that $r = \infty$ corresponds to $s = \infty$, i.e., that the integral defined by (6) is divergent (otherwise the situation is quite trivial). Then (6) shows that

$$\int^\infty |F| ds < \infty \quad (8)$$

holds if and only if

$$\int^\infty |F| f dr < \infty \quad (9)$$

does. On the other hand, it is readily verified from (7) that (9) is equivalent to (2). Hence, (2) means that (8) is satisfied.

It is known² that, if F is any continuous function satisfying (8), then $\phi(s) - \phi_0(s) \rightarrow 0$ and $d(\phi - \phi_0)/ds \rightarrow 0$, as $s \rightarrow \infty$, is a pair of asymptotic requirements which establishes a one-to-one correspondence between the solutions, $\phi = \phi(s)$, of (5) and the solutions, $\phi_0 = \phi_0(s)$, of the trivial approximation, $(d^2\phi_0/ds^2) + \phi_0 = 0$, to (5). Since this trivial differential equation admits of the solution $\phi_0(s) = e^{is}$, it follows that (5) has a solution satisfying the limit relation $\phi(s)/e^{is} \rightarrow 1$, where $s \rightarrow \infty$. Finally, it is seen from (6) that this limit relation is equivalent to (4) by virtue of the definition, $\psi = \phi/f^{\frac{1}{2}}$, of ϕ .

¹ The transformations applied to (1) can be interpreted as an adaptation of Liouville's substitution (as to the latter, see, e.g., E. L. Ince, *Ordinary Differential Equations* (Longmans Green, and Company, London, 1927), pp. 270-271).

² The result quoted is due to M. Bôcher, *Trans. Am. Math. Soc.* 1, 40 (1900). Today it has more than one proof very primitive in nature (see, for example, H. Weyl, *The Theory of Groups and Quantum Mechanics* (Methuen and Company, Ltd., London, 1931), pp. 71-73).

The Isotopic Composition of Normal Krypton and Xenon

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June 3, 1947

RECENTLY a mass-spectrometer investigation of the isotopes of krypton and xenon resulting from the fission of uranium-235 by thermal neutrons was reported.¹ This investigation was carried out with a 180°-deflection Nier-type mass spectrometer.² In the course of a parallel investigation with a 90°-deflection sector-type instrument,³ it was discovered that the measured abundance of krypton 80 in normal krypton was 10 percent greater than the value previously reported by Nier.⁴ Because of this discrepancy, it was decided to make a careful investigation of the isotopic composition of normal krypton and xenon. An automatic ion-current recording unit,⁵ which increases the precision of isotopic-abundance measurements, was used in this work.

Various possible systematic errors which might arise from resolution difficulties, mass discrimination, ion-current

discrimination, or secondary electron emission from the collector plate were investigated. These errors in the isotopic abundances were found to be less than one percent under our operating conditions. The purified gas samples were free from contamination, and there was no evidence of isotopic fractionation either during the course of sample preparation or during the passage of the gas through the capillary leak into the mass spectrometer tube.

Results for Krypton

Abundance data obtained from twenty recorded mass spectrograms of normal krypton using the 90° mass spectrometer and from ten such spectrograms using the 180° instrument are presented in Table I, along with the earlier data of Nier.⁴

TABLE I. Isotopic composition of normal krypton.

Mass unit	90° M.S.		180° M.S.		Nier*
	Abundance (percent)	Mean deviation	Abundance (percent)	Mean deviation	Abundance (percent)
78	0.343	±0.003	0.341	±0.0003	0.35
80	2.223	±0.009	2.223	±0.002	2.01
82	11.510	±0.040	11.490	±0.010	11.53
83	11.490	±0.030	11.470	±0.020	11.53
84	57.000	±0.090	57.040	±0.040	57.10
86	17.420	±0.030	17.440	±0.030	17.47

* See reference 4.

From this table it may be seen that the abundance data for krypton 80 from the present investigation is greater by ten percent than the value reported by Nier. Considering the good agreement between our 90° and 180° data, it would appear that Nier's value must have been in error. This fact was brought to the attention of Professor Nier, who found, on returning to his original records of 1937, that he had used the wrong shunt factor in computing the background correction for krypton 80.⁶ In Table II is presented the average of our 90° and 180° mass spectrometer abundance data for normal krypton, along with the corrected data of Nier.⁶ Table II gives the final results on the isotopic abundance of normal krypton.

TABLE II. Isotopic composition of normal krypton.

Mass unit	Present data (average) abundance (percent)	Nier's corrected data** abundance (percent)
78	0.342	0.346
80	2.228	2.261
82	11.500	11.500
83	11.480	11.500
84	57.020	56.950
86	17.430	17.430

** See reference 6.

From this table, the good agreement between Nier's corrected values and our abundance data for krypton 82, 83, 84, and 86 is evident. However, in the case of krypton 78 and 80, a discrepancy of about one percent is noted.

Results for Xenon

Abundance data from six complete recorded mass spectrograms of normal xenon using the 180° mass spectrometer are presented in Table III, in which Nier's earlier data are also included.⁴

TABLE III. Isotopic composition of normal xenon.

Mass unit	Present data (180° M.S.)		Nier's data*** abundance (percent)
	Abundance (percent)	Mean deviation	
124	0.095	±0.001	0.094
126	0.088	±0.001	0.088
128	1.917	±0.006	1.900
129	26.240	±0.080	26.230
130	4.053	±0.005	4.070
131	21.240	±0.030	21.170
132	26.930	±0.020	26.960
134	10.520	±0.020	10.540
136	8.930	±0.030	8.950

*** See reference 4.

The agreement between our results for xenon and those previously obtained by Nier is quite within the limits of accuracy claimed.

The authors are indebted to Dr. F. P. Lossing and Mr. R. B. Shields for assistance with the electronic units of the recorder and the mass spectrometers. The financial assistance of the National Research Council of Canada is gratefully acknowledged.

¹ Thode and Graham, Can. J. Research **A25**, 1 (1947).² Thode, Graham, and Ziegler, Can. J. Research **B23**, 40 (1945).³ Graham, Harkness, and Thode, J. Sci. Inst. **24**, 119 (1947).⁴ Nier, Phys. Rev. **52**, 933 (1937).⁵ Lossing, Shields, and Thode, Can. J. Research, in press.⁶ Private communication from A. O. C. Nier, April 2, 1947.

Note on the Barometric Coefficient of Cosmic-Ray Intensity

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August 7, 1947

AN analysis of the observations on cosmic-ray intensity was made by Duperier¹ in London to find the connection between surface pressure and cosmic-ray intensity. The high correlation coefficient of -0.87 was obtained between the hourly numbers of cosmic particles, averaged in groups of 24 hours, and the barograph readings at the station averaged over the same time intervals. It was also found that the barometric coefficient β , represented by the slope of the corresponding regression line, was 3.45 percent per cm mercury. The effects of absorption and decay have been separated by expressing the relation between the number of cosmic particles at ground level (N), the barograph reading at the station (B), and the height of the pressure level at which mesons are generated, (H), in the form

$$N - N_m = \mu(B - B_m) + \mu'(H - H_m),$$

where the subscript m refers to mean values μ represents the true absorption coefficient in air, and μ' the mean rate of decay of mesons. The true absorption coefficient in air was found to be 2.28 percent per cm mercury and the mean rate of decay of mesons 5.4 percent per km.

The purpose of the present note is to show that it is possible to find the value of the barometric coefficient by another method. We consider the effects of absorption and decay separately and shall first investigate the variation due to decay. Penner has analyzed the variations of pres-

sure at different levels over Sault Ste. Marie, Michigan, and a diagram showing the variations is reproduced in Haurwitz's book.² The conditions over Europe and North America are similar, and Penner's diagram may be regarded as representative for the north temperate latitudes in general. We take the meson-producing layer to be at a height of 16 km corresponding to 75-mm pressure. It is seen from Penner's diagram that the pressure at 16 km rises by 1 mb when the pressure at the ground level rises by 4 mb. The isobar corresponding to 75-mm pressure rises through a height $\delta p / g\rho$, where δp is the rise in pressure at 16 km and ρ is the density of air at that height. Hence when the pressure at 16 km rises by 1 mb, the isobar corresponding to 75-mm pressure rises through 52 meters.

The mean free path (decay) of mesons (L) is related to the lifetime of mesons at rest, τ_0 , by the well-known equation $L = E\tau_0/cM$, where M is the rest mass and E the energy of the mesons. If we assume that the mesons have a mean energy of 3×10^9 ev and take M equal to 200 times the mass of an electron and τ_0 equal to 2×10^{-6} sec. (Wilson),³ we have $L = 18$ km. If N_0 is the number of cosmic particles at 16 km, we see that $N_m = N_0 e^{-16/L}$. When the pressure level at which mesons are produced rises through 52 meters, $N = N_0 e^{-16.05/18}$, and the percentage variation is $(N - N_m)/N_m \times 100$, or neglecting the negative sign, 0.28 percent. Hence, we infer that the percentage variation per cm of mercury is 0.92. The variation of cosmic-ray intensity due to true absorption can be deduced from the measurements by Ehmert⁴ of the absorption curve in water, and is estimated by Duperier to be 1.65×10^{-3} cm²/g or 2.24 percent per cm of mercury. Therefore, the barometric coefficient is 3.16 percent per cm of mercury which is in good agreement with the value of 3.45 percent per cm of mercury obtained by Duperier.

¹ A. Duperier, Nature **153**, 529 (1944).² B. Haurwitz, *Dynamic Meteorology* (1941).³ J. G. Wilson, Science Progress **35**, 137 (1947).⁴ Ehmert, Zeits. f. Physik **106**, 751 (1937).

H³ and the Mass of the Neutrino

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July 28, 1947

THE maximum energy of the β -particles from H³ has most recently¹ been reported as 11 ± 2 kev. The unusually low energy of this β -spectrum makes it extremely sensitive as an indicator of a non-vanishing rest mass for the neutrino. Coupled with measurements of the H³ half-life, it shows that the neutrino cannot have a mass greater than 2 to 3 percent of the electron's rest mass. Moreover, the H³ decay rate as presently known seems 6 to 10 times too rapid in comparison with that of heavier elements unless the neutrino is attributed a finite mass of the magnitude mentioned.

The relation² between the half-life t and the maximum energy E_0 (in units of mc²; here $E_0 = 0.021_9$) is such that the product $[|M|^2 t f(E_0)]$ should be the same for all allowed β -transitions. $|M|^2$ is the so-called "nuclear matrix element" which measures the overlapping of the states